difference in stature was found to be 2·751 inches, and the standard deviation of its distribution 2·070 inches. The correlation between difference in stature and size of family was −0·0236, or greater fertility appears associated with small differences. The observations, however, are so few (205) that the probable error of the correlation is 0·0471, and thus no stress can be laid on this result. If the reader asks why is not the result in § 7 conclusive, the answer must be, it would be conclusive, if the means of the husbands and wives weighted with their fertility were the same as when they were unweighted; increased correlation would then necessarily connote that fertility was associated with homogamy. Actually the fact that absolutely taller mothers are the more fertile alters the centre of the correlation table, and somewhat obscures the issue as to whether the whole increase of correlation is really due to homogamy being correlated with fertility.

That in man, whether from conscious or unconscious sexual selection, there is far more homogamy than has hitherto been supposed, my family data cards amply demonstrate. If in man, then with great probability we can consider it to exist in other forms of life. But the existence of such homogamy is of immense importance for the problem of differentiation. The present statistics do not enable us to say whether homogamy in man is definitely correlated with fertility; they do show that fertility is not a random character, but depends upon the relative size of husband and wife, and thus bring evidence in favour of genetic selection. I can conceive no more valuable investigation than a series of experiments or measurements directed to ascertaining whether homogamy is or is not correlated with fertility, but such investigation, bearing in mind Darwin's conclusions, should carefully distinguish between exogamous and endogamous homogamy.

"On the Numerical Computation of the Functions $G_0(x)$, $G_1(x)$, and $J_n(x\sqrt{i})$." By W. Steadman Aldis, M.A. Communicated by Professor J. J. Thomson, F.R.S. Received and Read June 15, 1899.

1. The complete solution of the equation

$$\frac{d^2y}{dx^2}+\frac{1}{x}\cdot\frac{dy}{dx}-\left(1+\frac{n^2}{x^2}\right)y=0$$

may be written

$$y = AI_n(x)+BK_n(x),$$

where

$$I_n(x) = \sum_{r=0}^{r=\infty}\frac{(\frac{1}{2}x)^{n+2r}}{\Pi(r)\,.\,\Pi(n+r)} \qquad \ldots\ldots(1);$$

and, if n be any positive integer or zero,

$$K_n(x) = EI_n(x) - \Lambda_n(x) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2),$$

where

$$\begin{aligned}\Lambda_n(x) &= I_n(x)\log x \\ &+ \frac{(-2)^{n-1}\Pi(n-1)}{x^n}\left\{1 - \frac{(\frac{1}{2}x)^2}{\overline{n-1}\,.\,1} + \frac{(\frac{1}{2}x)^4}{\overline{n-1}\,.\,\overline{n-2}\,.\,1\,.\,2} + \ldots\ldots\ldots\right. \\ &\left. + \frac{(-1)^{n-1}(\frac{1}{2}x)^{2n-2}}{\{\Pi(n-1)\}^2}\right\} - \tfrac{1}{2}\sum_{r=0}^{r=\infty}\frac{(\frac{1}{2}x)^{n+2r}}{\Pi(r)\,\Pi(n+r)}(S_r + S_{n+r}) \quad \ldots\ldots(3),\end{aligned}$$

S_r denoting $1 + \frac{1}{2} + \frac{1}{3} + \ldots\ldots + \frac{1}{r}$, with the special case $S_0 = 0$, and E being $\log 2 + \frac{\Gamma'(1)}{\Gamma(1)}$.

2. When x is a real quantity, the function $I_n(x)$ increases from zero (or unity, when $n = 0$) to an infinitely large quantity, as x passes from zero to infinity, while $K_n(x)$ decreases numerically from infinity to zero under the same circumstances.

The values of the functions $K_0(x)$, $K_1(x)$ have been tabulated by the present writer, and published in the 'Proceedings,' for values of x at intervals of 0·1 from 0·1 to 12·0. The elements used in the calculation of the earlier half of these results are available for computing the values of $K_0(x)$ and $K_1(x)$ in some cases when x is a complex quantity.

If x be a pure imaginary $= zi$, z being a scalar, it is easily seen that

$$I_n(x) = i^n J_n(z) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4),$$

where $J_n(z)$ is the ordinary Bessel's function of the first kind and nth order.

If also $Y_n(z)$ denote Neumann's function of the nth order, and $G_n(z)$ be a function defined by the relation

$$G_n(z) = E\,.\,J_n(z) - Y_n(z) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5),$$

it can be shown without much difficulty that

$$K_n(x) = i^n G_n(z) - \frac{\pi}{2} i^{n+1} J_n(z) \quad \ldots\ldots\ldots\ldots\ldots\ldots (6).$$

3. The numerical calculation of the functions $G_0(x)$ and $G_1(x)$ can be made to depend on that of $K_0(x)$ and $K_1(x)$ for any values of x for which the convergent series (1) and (3) are applicable. In doing this it is necessary to calculate the elements of $J_0(x)$ and $J_1(x)$, and incidentally to compute these functions.

With the notation used in the writer's paper on the computation of $K_0(x)$ and $K_1(x)$, it is easily seen that

$$\mathrm{J}_0(x) = \beta_0-\beta_2+\beta_4-\beta_6+\ldots\ldots = \sum_{m=0}^{m=\infty}\beta_{4m}-\sum_{m=0}^{m=\infty}\beta_{4m+2} \quad\ldots\ldots(7),$$

$$\mathrm{J}_1(x) = \beta_1-\beta_3+\beta_5-\beta_7+\ldots = \sum_{m=0}^{m=\infty}\beta_{4m+1}-\sum_{m=0}^{m=\infty}\beta_{4m+3}\ldots\ldots\ldots(8).$$

Thus the elements β, used in the computation of $\mathrm{I}_0(x)$ and $\mathrm{I}_1(x)$, for any value of x can be easily used to derive the values of $\mathrm{J}_0(x)$ and $\mathrm{J}_1(x)$

4. We have further

$$\mathrm{G}_0(x) = \mathrm{J}_0(x)(\mathrm{E}-\log x)-\left\{\frac{(\frac{1}{2}x)^2}{\Pi(1)|^2}-\frac{(\frac{1}{2}x)^4}{\Pi(2)|^2}\mathrm{S}_2-\frac{(\frac{1}{2}x)^6}{\Pi(3)|^2}\mathrm{S}_3-\ldots\ldots\right\}$$

also $$0 = \mathrm{J}_0(x)-1+\left\{\frac{(\frac{1}{2}x)^2}{\Pi(1)|^2}-\frac{(\frac{1}{2}x)^4}{\Pi(2)|^2}+\frac{(\frac{1}{2}x)^6}{\Pi(3)|^2}-\ldots\ldots\right\}$$

whence by addition

$$\mathrm{G}_0(x) = \mathrm{J}_0(x)\{\mathrm{E}+1-\log x\}+\{\gamma_4-\gamma_6+\gamma_8-\ldots\ldots\}-1 \quad\ldots(9),$$

using the notation of the former paper.

Again,

$$\mathrm{G}_1(x) = \mathrm{J}_1(x)(\mathrm{E}-\log x)+\frac{1}{x}$$

$$+\tfrac{1}{2}\left\{\frac{x}{2}-\frac{(\frac{1}{2}x)^3(\mathrm{S}_1+\mathrm{S}_2)}{\Pi(1)\Pi(2)}+\ldots\ldots+\frac{(-1)^r(\frac{1}{2}x)^{2r+1}(\mathrm{S}_r+\mathrm{S}_{r+1})}{\Pi(r)\Pi(r+1)}+\ldots\right\}$$

but $$0 = \mathrm{J}_1(x)-\left\{\frac{x}{2}-\frac{(\frac{1}{2}x)^3}{\Pi(1)\Pi(2)}+\ldots\ldots+\frac{(-1)^x(\frac{1}{2}x)^{2r+1}}{\Pi(r)\,.\,\Pi(r+1)}+\ldots\ldots\right\};$$

whence, adding,

$$\mathrm{G}_1(x) = \mathrm{J}_1(x)\{\mathrm{E}+1-\log x\}+\frac{1}{x}-\frac{x}{4}$$

$$-\frac{(\frac{1}{2}x)^3(\mathrm{S}_1+\mathrm{S}_2-2)}{2\Pi(1)\Pi(2)}+\ldots\ldots+\frac{(-1)^r(\frac{1}{2}x)^{2r+1}(\mathrm{S}_r+\mathrm{S}_{r+1}-2)}{2\Pi(r)\Pi(r+1)}\ldots\ldots$$

But

$$\frac{(\frac{1}{2}x)^3(\mathrm{S}_1+\mathrm{S}_2-2)}{2\Pi(1)\Pi(2)} = \frac{(\frac{1}{2}x)^3(\mathrm{S}_1-1)}{\Pi(1)\Pi(2)}+\frac{1}{8}\left(\frac{x}{2}\right)^3 = \frac{\beta_4}{x}$$

....

$$\frac{(\frac{1}{2}x)^{2r+1}(\mathrm{S}_r+\mathrm{S}_{r+1}-2)}{2\Pi(r)\Pi(r+1)} = \frac{(\frac{1}{2}x)^{2r+1}(\mathrm{S}_r-1)}{\Pi(r)\Pi(r+1)}+\frac{(\frac{1}{2}x)^{2r+1}}{2\{\Pi(r+1)\}^2}$$

$$= \frac{\frac{x}{2}\gamma_{2r}}{r+1}+\frac{\beta_{2r+2}}{x}$$

Hence

$$G_1(x) = J_1(x)\{E+1-\log x\}+\frac{1}{x}-\frac{x}{4}$$
$$+\frac{x}{2}\left\{\frac{1}{3}\gamma_4-\frac{1}{4}\gamma_6+\frac{1}{5}\gamma_8-\frac{1}{6}\gamma_{10}+\ldots\right\}-\frac{1}{x}\{\beta_4-\beta_6+\beta_8-\ldots\}.$$

The last portion of this $= -\frac{1}{x}\{J_0(x)-1+\beta_2\} = -\frac{J_0(x)}{x}+\frac{1}{x}-\frac{x}{4}$,

whence $$G_1(x) = J_1(x)\{E+1-\log x\}+\frac{2}{x}-\frac{x}{2}$$
$$+\frac{x}{2}\left\{\frac{1}{3}\gamma_4-\frac{1}{4}\gamma_6+\frac{1}{5}\gamma_8-\frac{1}{6}\gamma_{10}+\ldots\right\}-\frac{J_0(x)}{x} \ldots\ldots (10)$$

5. The quantities β and γ, $\frac{\gamma_{2r}}{r+1}$, and the multiples of the different values of $(E+1-\log x)$ have been computed for the values of x, 0·1, 0·2,......, 6·0, in the process of calculation of $K_0(x)$ and $K_1(x)$, given in the writer's former paper. It has been, therefore, an easy matter to find by (7), (8), (9), and (10), the quantities $J_0(x)$, $J_1(x)$, $G_0(x)$, and $G_1(x)$ corresponding to the same values of x. The former two are of course well known, but the recalculation affords a valuable verification of the correctness of the quantities β. The results are given in Table I, appended to this paper, negative values being indicated by the use of old numeral type.

The formula used for verifying the values of I and K was

$$I_1(x) \,.\, K_0(x) - I_0(x) \,.\, K_1(x) = \frac{1}{x}.$$

Replacing x by zi, by means of (4) and (6), this gives

$$iJ_1(z)\left\{G_0(z)-\frac{\pi}{2}iJ_0(z)\right\}-J_0(z)\left\{iG_1(z)+\frac{\pi}{2}J_1(z)\right\} = \frac{1}{zi}$$

whence $$J_1(z) \,.\, G_0(z) - J_0(z) \,.\, G_1(z) = -\frac{1}{z} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (11).$$

This formula has been applied throughout Table I to each set of four values, calculated to three places beyond those given. Where the last figure has been increased by unity, in consequence of the first omitted figure being equal to or greater than five, the fact is indicated by a dot after the last figure. The column $G_0(x)$ has also been tested with satisfactory results by differencing.

6. The value of $I_n(x)$ can be readily expressed in terms of the quantities β, when n is either zero or unity, in one or two other cases, beside those of x, being a pure imaginary or wholly real.

For instance if $$x = z\epsilon^{\frac{1}{4}(i\pi)} = zi^{\frac{1}{2}},$$

then $$I_0(zi^{\frac{1}{2}}) = P_0 + Q_0 i, \text{ say,}$$

where $$P_0 = \beta_0 - \beta_4 + \beta_8 - \ldots\ldots \qquad Q_0 = \beta_2 - \beta_6 + \beta_{10} - \ldots\ldots$$

Thus the values of P_0 and Q_0 are easily deduced, and, therefore, that of $I_0(xi^{\frac{1}{2}})$.

The same process gives the value of $J_0(xi^{\frac{1}{2}})$, for,

since $$J_0(x) = \beta_0 - \beta_2 + \beta_4 - \beta_6 + \ldots\ldots$$

it is easily seen that

$$J_0(xi^{\frac{1}{2}}) = \beta_0 + \beta_2 i - \beta_4 + \beta_6 \dot{2} + \beta_8 - \ldots\ldots = P_0 - Q_0 i \ldots\ldots\ldots(12).$$

The values of P_0 and Q_0 are tabulated in the Report of the British Association for 1893, to nine places of decimals for intervals of 0·2 of a unit. Table II at the end of this paper gives them for the same number of places, and for the same intervals as have been used in the calculation of the K and G functions.

P_0 and Q_0 are denoted in the Table II by X and Y in accordance with the notation adopted by the Committee of the British Association, negative values being denoted by the use of old numeral type.

7. Assuming the accuracy of the values used for the quantities β, an accuracy guaranteed by the tests to which the Tables for I and K in the former paper have been subjected, the relation between I and J gives a very easy check for detecting and correcting any mistakes in addition or copying figures in finding the values of J.

Thus

$$I_0(x) = \sum_{m=0}^{m=\infty} \beta_{4m} + \Sigma\beta_{4m+2}$$

$$J_0(x) = \Sigma\beta_{4m} - \Sigma\beta_{4m+2}.$$

In finding $J_0(x)$, $\Sigma\beta_{4m}$ and $\Sigma\beta_{4m+2}$ are separately computed by addition of alternate terms from $I_0(x)$, and the smaller sum written down below the larger. In all cases in Table I the sum of these has first been taken, and the agreement or disagreement of this sum with the known correct value of $I_0(x)$ has shown either that there was no mistake, or has revealed where such mistake was committed, and secured its correction.

A similar test of accuracy in finding $J_1(x)$ is derived from the known values of $I_1(x)$.

In like manner, since

$$\Sigma\beta_{4m} = \Sigma\beta_{8m} + \Sigma\beta_{8m+4},$$

and $$X = P_0 = \Sigma\beta_{8m} - \Sigma\beta_{8m+4},$$

the known value of $\Sigma\beta_{4m}$, obtained in finding $J_0(x)$, gives a check on mistakes in calculating X. The known value of $\Sigma\beta_{4m+2}$ does the same service in regard to the computation of Q_0 or Y.

8. By formula (8)

$$J_1(x) = \beta_1 - \beta_3 + \beta_5 - \beta_7 + \ldots\ldots .$$

Hence
$$J_1(x\sqrt{i}) = \beta_1 i^{\frac{1}{2}} - \beta_3 i^{\frac{3}{2}} + \beta_5 i^{\frac{5}{2}} - \ldots\ldots$$

$$= \beta_1 \cos\frac{\pi}{4} - \beta_3 \cos\frac{3\pi}{4} + \beta_5 \cos\frac{5\pi}{4} - \ldots\ldots$$
$$+ i\left(\beta_1 \sin\frac{\pi}{4} - \beta_3 \sin\frac{3\pi}{4} + \beta_5 \sin\frac{5\pi}{4} \ldots\ldots\right)$$

$$= \frac{1}{\sqrt{2}}\{\beta_1 + \beta_3 - \beta_5 - \beta_7 + \beta_9 + \beta_{11} - \beta_{13} - \beta_{15} + \ldots\ldots$$
$$+ i(\beta_1 - \beta_3 - \beta_5 + \beta_7 + \beta_9 - \beta_{11} - \beta_{13} + \beta_{15} + \ldots\ldots)\}$$

$$= \frac{1}{\sqrt{2}}\{X_1 + Y_1 i\}\ldots\ldots, \text{ say} \quad \ldots\ldots \quad (13),$$

where

$$X_1 = \Sigma(\beta_{8m+1} + \beta_{8m+3}) - \Sigma(\beta_{8m+5} + \beta_{8m+7}),$$
$$Y_1 = \Sigma(\beta_{8m+1} + \beta_{8m+7}) - \Sigma(\beta_{8m+3} + \beta_{8m+5}),$$

the summation being in all cases from $m = 0$ to the largest value of m which gives sensible values for β.

The values of X_1, Y_1, computed by these formulæ from the known values of β, are given in Table II.

The computations evidently admit of a check to inaccuracy of the same nature as those given in the last article.

Another form of the values of X_1 and Y_1 is given by

$$X_1 = \Sigma(\beta_{8m+1} - \beta_{8m+5}) + \Sigma(\beta_{8m+3} - \beta_{8m+7}),$$
$$Y_1 = \Sigma(\beta_{8m+1} - \beta_{8m+5}) - \Sigma(\beta_{8m+3} - \beta_{8m+7}),$$

which reduces the computation to that of the two quantities

$$\Sigma(\beta_{8m+1} - \beta_{8m+5}) \text{ and } \Sigma(\beta_{8m+3} - \beta_{8m+7}),$$

so that if these be denoted by P_1 and Q_1

$$X_1 = P_1 + Q_1, \qquad Y_1 = P_1 - Q_1.$$

This form admits of somewhat different checks to mistakes. The values in Table II have been computed independently in the two ways, so that the writer has every confidence that they may be relied on as correct. The column for Y_1 has also been differenced with satisfactory results.

9. The well-known sequence laws

$$J_{n+1}(x) = \frac{2n}{x} J_n(x) - J_{n-1}(x) \dots\dots\dots\dots\dots\dots \quad (14),$$

$$dJ_0/dx = -J_1 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (15)$$

can be utilised, the former to obtain the values of $J_2(xi^{\frac{1}{2}})$, $J_3(xi^{\frac{1}{2}})$ ..., and the latter to give a verification to some extent of the values of $J_1(xi^{\frac{1}{2}})$, by means of the formulæ given in the writer's paper on I and K, which express dy/dx in terms of a series of equidistant values of y.

Thus, since

$$dJ_0/dx = -J_1,$$

replacing x by $xi^{\frac{1}{2}}$, and using the values already assumed for $J_0(xi^{\frac{1}{2}})$ and $J_1(xi^{\frac{1}{2}})$, it follows that

$$\frac{d(X - Yi)}{dx} = -i^{\frac{1}{2}} \cdot \frac{X_1 + Y_1 i}{\sqrt{2}} = -\frac{1+i}{\sqrt{2}} \cdot \frac{X_1 + Y_1 i}{\sqrt{2}}.$$

Whence

$$\left.\begin{aligned} dX/dx &= -\tfrac{1}{2}(X_1 - Y_1), \\ dY/dx &= \tfrac{1}{2}(X_1 + Y_1) \end{aligned}\right\} \dots\dots\dots\dots\dots\dots \quad (16).$$

By means of the formulæ (18), (19), and (21) of Articles 17—19 in the paper above referred to, this formula gives a check to the series of values in Table II to a considerable number of decimal places, to thirteen places with the last approximation.

10. For determining the values of $J_2(xi^{\frac{1}{2}})$, $J_3(xi^{\frac{1}{2}})$, ... by the sequence law, it is convenient to denote these quantities by the symbol $X_n + Y_n i$ when n is even, and by $\frac{1}{\sqrt{2}}(X_n + Y_n i)$ when n is odd. This will be found to avoid irrational multipliers in the successive derivations.

Equation (14), putting $xi^{\frac{1}{2}}$ for x, gives

$$J_{n+1}(xi^{\frac{1}{2}}) = \frac{2n}{xi^{\frac{1}{2}}} \cdot J_n(xi^{\frac{1}{2}}) - J_{n-1}(xi^{\frac{1}{2}}).$$

The cases of n odd and n even must be separately considered.

First let n be odd. The equation gives, remembering that $i^{-\frac{1}{2}} = \frac{1 - \dot{2}}{\sqrt{2}}$,

$$X_{n+1} + Y_{n+1} i = \frac{n}{x}(1 - i)(X_n + Y_n i) - (X_{n-1} + Y_{n-1} i).$$

Whence, if n be odd,

$$\left.\begin{aligned} X_{n+1} &= \frac{n}{x}(X_n + Y_n) - X_{n-1} \\ Y_{n+1} &= \frac{n}{x}(Y_n - X_n) - Y_{n-1} \end{aligned}\right\} \dots\dots\dots\dots\dots \quad (17).$$

If, secondly, n be even, the equation gives

$$\frac{1}{\sqrt{2}}(X_{n+1}+iY_{n+1}) = \frac{2n}{x}\frac{1-i}{\sqrt{2}}(X_n+Y_n i) - \frac{1}{\sqrt{2}}(X_{n-1}+Y_{n-1}i),$$

whence

$$\left.\begin{aligned} X_{n+1} &= \frac{2n}{x}(X_n+Y_n) - X_{n-1} \\ Y_{n+1} &= \frac{2n}{x}(Y_n-X_n) - Y_{n-1} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots \quad (18).$$

The most important special cases are when $n = 1$ and $n = 2$. In the first, remembering that $X_0 = X$, $Y_0 = -Y$ (Article 6), equations (17) give

$$\left.\begin{aligned} X_2 &= \frac{X_1+Y_1}{x} - X \\ Y_2 &= \frac{Y_1-X_1}{x} + Y \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots \quad (19).$$

In the second case (18) gives

$$\left.\begin{aligned} X_3 &= \frac{4}{x}(X_2+Y_2) - X_1 \\ Y_3 &= \frac{4}{x}(Y_2-X_2) - Y_1 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots \quad (20).$$

In these derivations no labour is involved, except that of addition of known quantities, and division by x.

Table I.

x.	$J_0(x)$.	$J_1(x)$.
0·1	0·997 501 562 066 040 032 281	0·049 937 526 036 241 997 556
0·2	0·990 024 972 239 576 390 818·	0·099 500 832 639 235 995 398·
0·3	0·977 626 246 538 296 087 570·	0·148 318 816 273 104 007 741
0·4	0·960 398 226 659 563 450 344	0·196 026 577 955 318 744 107·
0·5	0·938 469 807 240 812 904 228	0·242 268 457 674 873 886 384·
0·6	0·912 004 863 497 210 775 955·	0·286 700 988 063 915 739 746·
0·7	0·881 200 888 607 405 280 839·	0·328 995 741 540 058 947 849·
0·8	0·846 287 352 750 480 266 089	0·368 842 046 094 169 994 205
0·9	0·807 523 798 122 544 777 302	0·405 949 546 078 805 674 605·
1·0	0·765 197 686 557 966 551 450·	0·440 050 585 744 933 515 960·
1·1	0·719 622 018 527 511 015 975·	0·470 902 394 866 292 936 849·
1·2	0·671 132 744 264 362 673 475	0·498 289 057 567 215 480 211·
1·3	0·620 085 989 561 509 131 673	0·522 023 247 414 660 396 129·
1·4	0·566 855 120 374 288 721 361·	0·541 947 713 930 854 533 153·
1·5	0·511 827 671 735 918 128 749	0·557 936 507 910 099 641 990
1·6	0·455 402 167 639 380 713 311	0·569 895 935 261 680 370 013
1·7	0·397 984 859 446 103 491 142·	0·577 765 231 529 023 219 798
1·8	0·339 986 411 042 558 350 093	0·581 516 951 731 165 183 470
1·9	0·281 818 559 374 385 470 714·	0·581 157 072 713 434 072 686·
2·0	0·223 890 779 141 235 668 052·	0·576 724 807 756 873 387 202
2·1	0·166 606 980 331 990 326 602	0·568 292 135 757 038 668 540·
2·2	0·110 362 266 922 173 950 988·	0·555 963 049 819 063 939 102
2·3	0·055 539 784 445 601 963 144	0·539 872 532 604 313 665 317
2·4	0·002 507 683 297 243 813 015	0·520 185 268 181 931 033 964
2·5	0·048 383 776 468 197 996 327	0·497 094 102 464 274 038 011·
2·6	0·096 804 954 397 038 249 909	0·470 818 266 517 578 669 733·
2·7	0·142 449 370 046 011 821 820	0·441 601 379 118 253 106 422
2·8	0·185 036 033 364 387 324 596	0·409 709 246 852 288 741 579
2·9	0·224 311 545 791 968 114 187·	0·375 427 481 813 095 896 391·
3·0	0·260 051 954 901 933 437 624	0·339 058 958 525 936 458 926·
3·1	0·292 064 347 650 697 540 058	0·300 921 133 101 057 626 662
3·2	0·320 188 169 657 122 907 289	0·261 343 248 780 504 837 363·
3·3	0·344 296 260 398 884 637 389	0·220 663 452 985 241 082 698
3·4	0·364 295 596 762 000 469 831·	0·179 225 851 681 507 110 994
3·5	0·380 127 739 987 263 377 379·	0·137 377 527 362 327 185 716
3·6	0·391 768 983 700 797 768 519·	0·095 465 547 177 876 403 846·
3·7	0·399 230 203 371 191 105 766	0·053 833 987 745 461 864 015
3·8	0·402 556 410 178 564 169 319	0·012 821 002 926 731 627 029
3·9	0·401 826 014 887 639 905 035·	0·027 244 039 620 779 926 253
4·0	0·397 149 809 863 847 372 287·	0·066 043 328 023 549 136 143
4·1	0·388 669 679 835 853 683 029	0·103 273 257 747 338 701 790·
4·2	0·376 557 054 367 567 663 516·	0·138 646 942 126 046 167 310·
4·3	0·361 011 117 236 535 112 103	0·171 896 560 221 540 474 678·
4·4	0·342 256 790 003 885 614 439·	0·202 775 521 923 086 594 695
4·5	0·320 542 508 985 121 424 355	0·231 060 431 923 370 634 008
4·6	0·296 137 816 574 141 142 650·	0·256 552 836 097 444 561 708·
4·7	0·269 330 789 419 752 826 396	0·279 080 735 843 335 330 140
4·8	0·240 425 327 291 183 452 194·	0·298 499 858 099 557 876 149·
4·9	0·209 738 327 585 326 314 755·	0·314 694 671 015 190 603 203·
5·0	0·177 596 771 314 338 304 347	0·327 579 137 591 465 222 038·
5·1	0·144 334 747 060 500 516 529	0·337 097 202 018 231 840 465·
5·2	0·110 290 439 790 986 539 621	0·343 223 005 871 921 903 218·
5·3	0·075 803 111 585 584 160 063·	0·345 960 833 801 186 199 542
5·4	0·041 210 101 244 991 307 084·	0·345 344 790 779 586 326 575
5·5	0·006 843 869 417 819 196 824·	0·341 438 215 429 043 350 180·
5·6	0·026 970 884 685 114 476 356·	0·334 332 836 291 007 483 208
5·7	0·059 920 009 724 037 401 926·	0·324 147 680 222 856 214 217
5·8	0·091 702 567 574 816 188 248	0·311 027 744 303 942 414 148
5·9	0·122 033 354 592 822 673 484·	0·295 142 444 729 016 123 857·
6·0	0·150 645 257 250 996 931 662	0·276 683 858 127 565 608 173·

N.B.—Negative quantities are

Table I.

$G_0(x)$.	$G_1(x)$.	x.
2·409 976 437 967 912 294 552	10·145 696 654 505 820 445 994	0·1
1·698 196 269 260 531 005 616·	5·221 052 082 235 180 455 883·	0·2
1·268 062 370 733 913 360 785	3·602 001 128 335 204 510 007	0·3
0·951 941 166 032 609 089 045·	2·797 387 265 631 115 266 589	0·4
0·698 248 393 783 854 194 778	2·311 383 429 386 515 572 834	0·5
0·484 606 170 757 539 963 706·	1·979 818 098 470 311 722 022	0·6
0·299 495 770 651 788 694 072·	1·732 980 846 329 450 701 757·	0·7
0·136 348 702 042 021 281 732·	1·536 465 279 810 038 555 299	0·8
0·008 840 923 388 656 204 883·	1·371 504 028 549 382 729 782	0·9
0·138 633 715 204 053 999 681	1·227 126 230 143 571 489 243	1·0
0·254 725 363 498 849 106 693·	1·096 603 640 617 960 561 767	1·1
0·358 272 729 071 792 761 119	0·975 678 743 748 170 263 436·	1·2
0·450 088 686 532 541 263 114	0·861 612 777 028 331 588 570·	1·3
0·530 764 428 542 739 360 172·	0·752 642 307 119 771 213 489·	1·4
0·600 749 364 688 180 915 674	0·647 652 876 756 467 095 194	1·5
0·660 405 024 575 635 605 540	0·545 974 258 657 556 467 494	1·6
0·710 042 351 497 427 739 063·	0·447 246 939 888 742 173 248·	1·7
0·749 947 984 061 056 004 754·	0·351 331 953 359 999 750 701	1·8
0·780 402 985 970 970 798 205·	0·258 247 983 282 347 884 849·	1·9
0·801 696 231 883 694 215 426·	0·168 126 150 312 430 935 228·	2·0
0·814 133 899 087 413 666 664	0·081 176 574 108 327 549 604	2·1
0·818 046 042 540 011 194 399·	0·002 337 013 951 404 941 779	2·2
0·813 790 929 365 743 495 271·	0·082 117 015 702 804 981 444·	2·3
0·801 757 612 346 090 680 037	0·157 847 655 213 986 366 030	2·4
0·782 367 091 369 019 468 035·	0·229 207 675 130 978 077 462	2·5
0·756 072 323 668 009 246 676	0·295 880 763 567 315 512 986·	2·6
0·723 357 283 363 643 701 757	0·357 564 209 833 291 131 344	2·7
0·684 735 229 033 948 656 974	0·413 976 136 265 745 314 572·	2·8
0·640 746 308 772 709 085 294	0·464 861 550 729 216 627 288	2·9
0·591 954 611 480 711 143 919·	0·509 997 393 867 205 323 674·	3·0
0·538 944 758 310 761 413 627	0·549 196 706 485 298 624 293	3·1
0·482 318 117 438 413 367 641·	0·582 312 008 724 774 045 611·	3·2
0·422 688 717 401 572 140 599	0·609 237 959 388 677 935 217	3·3
0·360 678 928 264 659 951 052	0·629 913 347 832 572 686 549·	3·4
0·296 914 975 194 465 215 873·	0·644 322 460 111 513 733 523	3·5
0·232 022 345 240 471 206 452·	0·652 495 854 105 516 078 102·	3·6
0·166 621 144 872 495 980 327	0·654 510 574 081 292 622 136	3·7
0·101 321 462 912 008 607 214·	0·650 489 832 836 477 094 620	3·8
0·036 718 790 734 042 399 784	0·640 602 188 665 708 742 081	3·9
0·026 610 451 105 001 945 410	0·625 060 244 480 341 903 495·	4·0
0·088 113 233 426 177 404 311	0·604 118 897 200 782 671 826·	4·1
0·147 264 042 657 322 775 155	0·578 073 166 790 814 300 183·	4·2
0·203 568 768 228 724 991 685	0·547 255 635 837 897 775 759·	4·3
0·256 568 315 804 579 439 611·	0·512 033 532 262 535 404 617·	4·4
0·305 841 912 363 794 369 172·	0·472 805 489 452 910 279 056·	4·5
0·351 010 072 657 030 554 463	0·429 998 019 780 139 344 891·	4·6
0·391 737 200 240 423 447 540·	0·384 061 738 984 071 875 442	4·7
0·427 733 800 104 815 454 258	0·335 467 380 273 038 857 255	4·8
0·458 758 283 862 071 400 448·	0·284 701 638 108 088 275 409	4·9
0·484 618 352 492 666 714 073	0·232 262 882 507 286 221 237·	5·0
0·505 171 945 773 571 858 198·	0·178 656 785 280 977 272 574	5·1
0·520 327 751 662 450 130 166	0·124 391 899 873 886 134 313	5·2
0·530 045 273 081 540 584 629·	0·069 975 236 430 273 453 649	5·3
0·534 334 453 689 368 500 921	0·015 907 873 305 342 552 148·	5·4
0·533 254 868 317 059 586 380·	0·037 319 354 483 812 230 517·	5·5
0·526 914 487 744 863 636 272	0·089 230 050 440 030 414 040·	5·6
0·515 468 031 370 191 891 880·	0·139 366 297 368 738 402 285	5·7
0.499 114 925 038 474 697 901·	0·187 292 507 445 744 860 142	5·8
0·473 096 884 841 038 443 599	0·232 599 047 277 909 357 025	5·9
0·452 695 151 000 080 566 867	0·274 905 605 978 175 743 452·	6·0

denoted by old numeral type.

Table II.

x.	$J_0(x\sqrt{}1) = X - Yi$	
	X	Y
0·1	0·999 998 437 500 067 816 840·	0·002 499 999 565 972 229 004·
0·2	0·999 975 000 017 361 109 182	0·009 999 972 222 229 166 666
0·3	0·999 873 437 944 946 038 780	0·022 499 683 594 150 451 545
0·4	0·999 600 004 444 436 543 214·	0·039 998 222 229 333 326 883
0·5	0·999 023 463 990 838 255 555·	0·062 493 218 382 199 458 650
0·6	0·997 975 113 905 224 846 398·	0·089 979 750 410 060 617 063·
0·7	0·996 248 828 444 070 123 287	0·122 448 938 981 613 810 260
0·8	0·993 601 137 745 414 585 178	0·159 886 229 503 894 323 928
0·9	0·989 751 356 659 594 009 089	0·202 269 363 489 470 399 618
1·0	0·984 381 781 213 086 883 966·	0·249 566 040 036 659 721 419
1·1	0·977 137 973 163 994 306 095·	0·301 731 269 206 265 863 908·
1·2	0·967 629 155 801 133 528 979·	0·358 704 419 873 150 681 448
1·3	0·955 428 746 808 400 572 511	0·420 405 965 634 100 168 746
1·4	0·940 075 056 652 724 712 846	0·486 733 933 588 908 060 448·
1·5	0·921 072 183 546 255 764 122	0·557 560 062 303 086 694 894
1·6	0·897 891 138 567 705 276 346	0·632 725 677 031 398 154 882·
1·7	0·869 971 236 987 757 520 821·	0·712 037 292 354 219 242 730·
1·8	0·836 721 794 210 160 854 515·	0·795 261 954 775 658 372 738·
1·9	0·797 524 166 991 521 789 701	0·882 122 340 574 509 297 036
2·0	0·751 734 182 713 808 228 551·	0·972 291 627 306 661 206 104
2·1	0·698 685 001 425 635 398 101·	1·065 388 160 849 286 232 192·
2·2	0·637 690 457 109 552 833 002	1·160 969 943 770 221 785 831·
2·3	0·568 048 926 137 096 187 234	1·258 528 975 115 816 306 932·
2·4	0·489 047 772 101 826 069 086·	1·357 485 476 450 273 287 287
2·5	0·399 968 417 129 531 339 957	1·457 182 044 159 804 184 047·
2·6	0·300 092 090 306 787 850 787·	1·556 877 773 663 311 509 857
2·7	0·188 706 303 992 608 423 524	1·655 742 407 252 085 252 722
2·8	0·065 112 108 427 346 531 305	1·752 850 563 814 438 038 253
2·9	0·071 367 825 831 445 002 541·	1·847 176 115 683 253 092 922·
3·0	0·221 380 249 598 693 888 868	1·937 586 785 266 042 766 897·
3·1	0·385 531 454 977 281 413 314·	2·022 839 041 963 733 753 825
3·2	0·564 376 430 484 566 549 458·	2·101 573 388 135 250 371 321
3·3	0·758 407 012 072 785 084 982	2·172 310 131 492 460 325 998·
3·4	0·968 038 995 314 976 506 884·	2·233 445 750 279 040 972 132·
3·5	1·193 598 179 589 928 060 082·	2·283 249 966 853 914 618 212
3·6	1·435 305 321 718 847 744 816·	2·319 863 654 812 663 506 793·
3·7	1·693 259 984 269 599 885 400	2·341 297 714 476 542 058 301·
3·8	1·967 423 272 739 419 648 007	2·345 433 061 385 529 680 393·
3·9	2·257 599 466 142 987 708 599	2·330 021 882 265 074 524 014·
4·0	2·563 416 557 258 579 754 134·	2·292 690 322 699 299 833 586·
4·1	2·884 305 732 008 850 753 468·	2·230 942 780 326 965 102 027
4·2	3·219 479 832 260 939 763 946·	2·142 167 986 657 022 889 923·
4·3	3·567 910 862 806 221 604 427·	2·023 647 069 440 171 807 909
4·4	3·928 306 621 502 089 386 988·	1·872 563 795 777 954 293 134
4·5	4·299 086 551 599 756 238 427	1·686 017 203 632 139 319 953·
4·6	4·678 356 937 208 980 936 827·	1·461 036 835 928 036 069 728
4·7	5·063 885 5[illegible]6 719 503 887 521·	1·194 600 796 822 301 663 253·
4·8	5·453 076 174 855 458 180 119	0·883 656 853 707 154 174 111
4·9	5·842 942 441 915 628 551 218	0·525 146 810 908 826 889 589·
5·0	6·230 082 478 666 357 733 185	0·116 034 381 550 200 378 097
5·1	6·610 653 3[illegible]7 304 570 918 646	0·346 663 217 591 247 641 801
5·2	6·980 346 402 874 876 505 440·	0·865 839 727 484 430 267 303·
5·3	7·334 363 435 462 957 925 254·	1·444 260 150 604 921 519 731
5·4	7·667 394 351 327 397 532 141	2·084 516 693 093 664 203 000
5·5	7·973 596 450 774 417 438 658·	2·788 980 154 734 066 597 920·
5·6	8·246 575 961 893 122 136 086	3·559 746 593 355 732 201 313·
5·7	8·479 372 252 085 205 623 568	4·398 579 111 649 335 813 378·
5·8	8·664 445 263 435 904 450 574	5·306 844 640 335 221 439 301·
5·9	8·793 666 753 132 378 304 231	6·285 445 622 573 310 185 248
6·0	8·858 315 966 045 036 088 551	7·334 746 540 847 962 419 331·

N.B.—Negative quantities are

Table II.

$$J_1(x\sqrt{i}) = \frac{1}{\sqrt{2}}(X_1 + Y_1 i)$$

X_1	Y_1	
0·050 062 473 952 908 664 336	0·049 937 473 963 759 358 667	**0·1**
0·100 499 165 972 569 560 158·	0·099 499 167 361 458 217 565	0·2
0·151 681 160 023 124 019 598	0·148 306 183 753 572 747 019	0·3
0·203 973 244 622 459 033 349·	0·195 973 422 399 762 737 375·	0·4
0·257 730 697 263 717 955 586	0·242 106 544 968 702 488 384	**0·5**
0·313 295 988 104 834 119 210	0·286 299 025 563 828 010 431·	0·6
0·370 995 377 179 567 178 556·	0·328 129 312 969 530 045 753	0·7
0·431 135 380 394 708 350 508·	0·367 158 134 979 370 769 977	0·8
0·493 999 080 858 644 866 754	0·402 925 975 846 922 443 874·	0·9
0·559 842 263 647 128 287 093·	0·434 950 759 289 066 366 621	**1·0**
0·628 889 353 965 588 933 103·	0·462 725 771 056 724 536 018·	1·1
0·701 329 140 828 180 143 887·	0·485 717 856 852 007 564 806	1·2
0·777 310 270 858 051 149 439·	0·503 365 933 293 808 160 710	1·3
0·856 936 499 638 710 654 481·	0·515 079 851 693 132 327 170	1·4
0·940 261 691 231 561 790 777	0·520 239 656 568 985 033 820·	**1·5**
1·027 284 560 039 561 065 790	0·518 195 283 086 146 165 860	1·6
1·117 943 153 161 996 231 566·	0·508 266 739 894 923 728 550	1·7
1·212 109 075 771 521 480 011	0·489 744 826 163 507 446 244	1·8
1·309 581 466 872 838 661 198·	0·461 892 433 875 541 892 607·	1·9
1·410 080 738 093 475 393 276	0·423 946 488 674 597 149 781	**2·0**
1·513 242 093 930 976 259 500	0·375 120 584 624 677 257 030	2·1
1·618 608 858 156 316 276 466	0.314 608 370 168 666 686 818·	2·2
1·725 625 637 867 629 817 384	0·241 587 744 246 948 343 429	2·3
1·833 631 364 016 387 962 836	0·155 225 922 923 833 261 156	2·4
1·941 852 255 102 137 149 639·	0·054 685 437 892 511 240 814	**2·5**
2·049 394 759 160 608 253 323·	0·060 868 871 182 468 533 464	2·6
2·155 238 538 158 111 742 033	0·192 261 808 450 176 681 909	2·7
2·258 229 568 452 541 032 652	0·340 298 656 269 695 509 185	2·8
2·357 073 441 082 328 838 113·	0·505 754 831 903 598 931 394·	2·9
2·450 328 956 287 265 629 836	0·689 364 308 171 542 723 711	**3·0**
2·536 402 117 829 866 100 448·	0·891 806 742 577 774 109 948·	3·1
2·613 540 644 345 464 751 601	1·113 693 263 253 173 849 031·	3·2
2·679 829 127 066 822 862 186·	1·355 550 864 962 340 196 186·	3·3
2·733 184 975 798 009 493 162	1·617 805 374 530 433 355 956·	3·4
2·771 355 307 894 804 733 452	1·900 762 952 457 942 309 529	**3·5**
2·791 914 948 174 775 185 144	2·204 590 106 332 283 224 110·	3·6
2·792 265 721 046 090 235 560·	2·529 292 202 033 743 119 928·	3·7
2·769 637 229 612 254 898 369·	2·874 690 470 791 989 051 438·	3·8
2·721 089 329 966 841 647 382·	3·240 397 524 001 181 798 218	3·9
2·643 516 522 207 920 785 664	3·625 791 403 469 366 015 499	**4·0**
2·533 654 492 728 293 864 014·	4·029 988 212 582 420 082 983	4·1
2·388 089 054 907 953 209 681	4·451 813 393 822 667 265 913·	4·2
2·203 267 747 262 508 230 792·	4·889 771 740 310 993 322 800·	4·3
1·975 514 359 177 636 802 071	5·342 016 253 646 428 366 716	4·4
1·701 046 664 354 870 983 541	5·806 315 987 397 841 481 769·	**4·5**
1·375 997 650 755 172 598 883·	6·280 023 045 247 471 131 240·	4·6
0·996 440 542 876 983 259 127·	6·760 038 935 071 492 457 005·	4·7
0·558 417 917 343 477 630 173·	7·242 780 515 229 332 115 515·	4·8
0·057 975 215 673 352 365 710·	7·724 145 807 063 384 231 105	4·9
0·508 801 041 580 956 398 748·	8·199 479 988 105 265 446 223	**5·0**
1·145 740 037 801 321 482 227·	8·663 541 923 740 282 796 986	5·1
1·856 538 144 879 196 341 558	9·110 471 641 066 744 879 434·	5·2
2·644 704 823 723 608 609 532·	9·533 759 197 337 575 219 035	5·3
3·513 502 669 101 221 083 206	9·926 215 446 588 907 115 423·	5·4
4·465 881 346 006 416 880 626·	10·279 945 261 704 303 219 887	**5·5**
5·504 405 188 780 265 648 900	10·586 323 825 049 679 446 625·	5·6
6·631 174 263 958 089 750 823·	10·835 976 658 709 369 731 829·	5·7
7·847 738 733 862 387 803 316·	11·018 764 124 970 370 273 237·	5·8
9·155 006 400 820 864 395 768	11·123 771 188 692 843 678 508·	5·9
10·553 143 362 143 207 874 207·	11·139 303 295 159 365 990 786·	**6·0**

denoted by old numeral type.